Mathematics of Computing
Mid Semester (Max Marks 40, Max Time 3h)

Indian Statistical Institute, Bangalore

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Q1. (1×10=10) True or false:

i If $L_1 = L_2 = \{\epsilon\}$ then the language formed by their concatenation: $L_1 \cdot L_2$ is not empty and is regular.

ii If L is a CFG with a deterministic PDA, then L is regular.

iii Regular languages have fewer strings in them than non-regular languages.

iv Every finite set of strings is regular.

v A DFA accepts L. We can modify it to accept L^* by making the initial state a final state and adding ϵ labeled edge from current final states to the initial state.

vi If a CFG produces a language L, the L is non-regular.

vii The well known CFL $\{a^n b^n\}$ is a subset of another language L. Therefore L cannot be regular.

viii The following grammar is regular:
$S \to aA$
$A \to Sb|b$

ix The language ww where $w \in \{0,1\}^*$ is regular.

x There are NFAs that recognize languages that DFAs cannot.

Q2. (3+3+4=10)

(a) If L is a regular language on $\{a, b\}$ show that $L^c = \{w, w \notin L\}$ is regular.

(b) Prove or disprove that if L_1 and L_2 are regular then the language $L_1 \backslash L_2$ (ie set difference) is regular.

(c) Consider the language L is the set of all strings of the form $a^i b^i c^i, i \geq 0$. Show that L is not regular.

Q3. (2+3=5) Let L be the language of strings that have exactly two zeros or exactly two ones. Thus strings like $011001 \notin L$, while $110 \in L$ and $0110 \in L$.

(a) Give the regular expression for L.

(b) Construct a DFA for the language.

Q4. (3+2=5) Consider the language of well formed parenthesized expressions. For example "(()())()()" is well formed, but not ")()("

(a) Write a CFG for the language.

(b) For your grammar, give a parse tree for the string "(()())()()"

Q5. (3+3=6)Consider the language that has all strings with equal number of 0s and 1s.

(a) Draw a PDA to recognize L.

(b) Write a CFG for L. You may, if you wish, use ideas from the PDA to inspire you.

Q6. (4)Consider the following language: $L = \{a^n b^{2n} c^{3n}, n \geq 0\}$. Is it context free? If yes, construct a CFG. Else prove it is not context free using the pumping lemma.